PART 3

TRADE THEORIES AND THE REAL WORLD TRADE PATTERNS

5. A Global Equilibrum Pattern of Specialization: A Model to Approximate Linder's World of Production and Trade

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A GLOBAL EQUILIBRIUM PATTERN OF SPECIALIZTION: A MODEL TO APPROXIMATE LINDER'S WORLD OF PRODUCTION AND TRADE*

Summary

A recent Swedish author, S. B. Linder, has introduced a novel and in a sense quite unorthodox approach to explain the pattern of production and trade: the representative demand pattern determines the range of goods that can be produced with comparative advantage and hence we would be able to forecast the pattern of production and trade on the basis of internal demand patterns of the countries.

Unfortunately, Linder has not rigorously formulated his analytical framework and has left his ingenious idea hanging in the air without chiselling it down into a theory. The main purpose of this paper is to formulate a theoretical framework which can specify the pattern of production and trade on the basis of internal demand pattern using the traditional Heckscher-Ohlin model and to incorporate the essential of Linder's seemingly unorthodox idea into the model. We will obtain a synthesized model which can predict the precise pattern of production and trade by specifying the internal demand pattern and which can also approximate Linder's world as closely as possible within the framework of the traditional international trade theory. We will see that the world envisaged by Linder is not as unorthodox as it sounds.

We will introduce a simple assumption which seems reasonably realistic that, if the total value of output is the same, each country has a tendency to minimize international transaction activities. With this assumption, it will be shown that we can eliminate the uncertainty concerning the pattern of production and trade of each country, and at the same time we can have a theoretical framework to specify the precise pattern of production and trade of countries on the basis of their internal demand patterns. After finishing the

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basic model, we will introduce a boundary condition related with the magnitude of internal demand for each good to incorporate essential of Linder's idea and investigate its implications on the pattern of production and trade of each country.

1. INTRODUCTION

Bhagwati, in his survey of the pure theory of international trade, introduces a novel and in a sense quite unorthodox approach to explain the pattern of production and trade attributable to S. B. Linder.[1] Linder develops his reasoning in the following fashion.[2] In a world of "imperfect knowledge," entrepreneurs will first produce goods for "domestic needs" of which "they are aware." As a successful firm grows, the "trade horizon" of the firm will be extended "across national frontiers." And then we come to his central hypotheses: (1) the precondition for a good to be produced domestically is the presence of "home demand"; (2) in order for a good to emerge as a potential export product, the internal demand for the good should be "representative";[3] and (3) since the representative demand pattern determines the range of goods that can be produced with comparative advantage, we would be able to forecast the pattern of production and trade on the basis of internal demand patterns of the countries.[4]

Unfortunately, as Bhagwati says, Linder has not rigorously formulated his analytical framework and has left his ingenious idea hanging in the air without chiselling it down into a theory. The main purpose of this paper is to formulate a theoretical framework which can specify the pattern of

[1]J. Bhagwati, "The Pure Theory of International Trade: A Survey," in *The Surveys of Economic Theory: II* published by the American Economic Association and the Royal Economic Society (New York: St. Martin's Press, 1965), pp. 182-184.

[2]S. B. Linder, *An Essay on Trade and Transformation* (New York: John Wiley and Sons, 1961).

[3]He seems to say that the demand for a good is "representative" if the internal demand for it is large enough or if the good typically fits into the standard of living attained by broad numbers of population, and that it is the good which has the representative demand that would likely be produced with comparative advantage because the producer should be familiar with the conditions under which his product will be used, and also because the large amount of "country's own needs" would be "the mother not only of innovation but also of invention," *ibid*., pp. 87-91.

[4]"If we make far-reaching use of 'representative demand' as a factor determining the range of potential exports, a country may export all its exportables and import goods with unrepresentative demand," *ibid*., p. 102.

production and trade on the basis of internal demand pattern using the traditional Heckscher-Ohlin model and to incorporate the essential of Linder's seemingly unorthodox idea into the model. We will obtain a synthesized model which can predict the precise pattern of production and trade by specifying the internal demand pattern and which can also approximate Linder's world as closely as possible within the framework of the traditional Heckscher-Ohlin theory of international trade. We will see that the world envisaged by Linder is not as unorthodox as it sounds.

We will assume in this paper a kind of perfect Heckscher-Ohlin world where the factor prices are equalized in every country, i.e., we will assume a world in which all the conditions which are necessary to result in factor-price-equalization are met. The assumption may seem far removed from Linder's thought, but with additional assumptions to be introduced in Section 3 and 4, the world approximated by our model comes very closely to the Linder's. Basically, Linder's world is regarded in this paper as a possible subset of the worlds which could be generated by the Heckscher-Ohlin theory of factor-price-equalization.

As is well known, factor prices will not be equalized if the number of factors (m) exceeds the number of goods (n). Factor prices can be equalized if $m \leq n$. Since $m = n$ is too trivial a case, we will consider only the general case of $m < n$. However, in the Heckscher-Ohlin model of factor price equalization, the precise degree of international specialization is indeterminate if $m < n$. The factor endowments and the demand patterns of each individual country have no formal places in the model itself to determine the precise pattern of production and trade of each country. In Section 3, we will introduce a simple assumption which seems reasonably realistic that, if the total value of output is the same, each country has a tendency to minimize interna-tional transaction activities. With this assumption, it will be shown that we can eliminate the uncertainty concerning the pattern of production and trade of each country, and at the same time we can have a theoretical framework to specify the precise pattern of production and trade of countries on the basis of their internal demand patterns. After finishing the basic model, we will introduce in Section 4 an assumption related with production and the magnitude of internal demand for each good to incorporate the essential of Linder's idea and investigate its implications on the pattern of production and trade of each country.

2. DESCRIPTION OF THE MODEL[5]

A productive process is defined as the set of all sums of the basic activities in which a given list of inputs and outputs appears.[6] Any efficient activity can be the basis of the production for a sector. Since only relative quantities matter, because of linearity, it is convenient to normalize the activity vectors; we shall select the *m*-dimensional row vector a^i for which

$$\sum_{j=1}^{m} a_j^i = 1$$

to represent the unit level of *i*th productive activity ($i = 1, . ., n$; $j = 1, ..., m$). Then any vector of factor inputs belonging to the activity can be obtained by multiplying the normalized vector by the appropriate nonnegative constant q_t. That is, $q_i a^i$ where the superscript *i* represents *i*th industry such that $a^i = (a_1^i, \ldots, a_m^i)$. The *n* by *m* matrix of coefficients for ultimate inputs (a_j^i) is written as *A*.

Let an *m*-dimensional row vector f^k represent the factor endowments of *k*th trading country. (The superscript *k* represents *k*th country where $k = 1, ... ,$ *N*.) We select the vector for which

$$\sum_{j=1}^{m} r_j^k = 1$$

to represent the unit level of factor endowments in the *k*th country. Then f^k can be obtained by multiplying the normalized vector by the appropriate constant B^k, i.e., $f^k = B^k r^k$ where $r^k = (r_1^k, \ldots, r_m^k)$.

Let *S* be the set of factor input vectors a^i for $i = 1, ... , n$ which satisfy the normalization rule

$$\sum_{j=1}^{m} a_j^i = 1.$$

[5]L. W. McKenzie proves the Heckscher-Ohlin theory of factor price equalization using the activity analysis approach, and the basic model of this paper is based on McKenzie's activity analysis in his "Equality of Factor Prices in World Trade," *Econometrica*, vol. 23, no. 3 (July 1955), pp. 239-257.

[6]A productive activity may use both goods and factors as inputs; but the intermediate goods are converted into ultimate primary factors and hence here an activity is represented by a vector of primary inputs only. Absence of joint production is also assumed in this paper.

Here r^k is also the element of S. All factor input vectors lie in S, and for a given set of good and factor prices (p, w) which is compatible with the profit conditions, we can consider the set K_{pw} of all elements of S which are expressible as positive linear combinations of a^i's coming from activities which earn zero profits at (p, w).[7] Here, z is in K_{pw} if and only if there are a^i,

$$r = \sum_{i=1}^{n} t_i a^i \quad \text{for} \quad 0 \le t_i \quad \text{and} \quad \sum_{i=1}^{n} t_i = 1$$

whose activities earn zero profits at (p, w) and
where t_i are the levels at which the productive activities in use in the equilibrium are operated. If it is to be possible to exhaust the given vector of factor supplies of the kth country, r^k must lie in K_{pw}. The points of K_{pw} which can be expressed as positive linear combinations of at least m linearly independent a^i's lie in the interior of K_{pw}. The competitive equilibrium with equalized factor prices, which is assumed throughout this paper, implies that

$$r = \sum_{i=1}^{n} t_i a^i \text{æ} \quad \text{for} \quad 0 \le t_i \quad \text{and} \quad \sum_{i=1}^{n} t_i = 1 \text{ö}$$

r^k lies in the interior of K_{pw} for all k so that
in every country without specialization in any country.[8]

If the number of productive activities a^i is equal to the number of factors, there is only one t vector in a country which can exhaust the given vector of factor supplies of the country, i.e., there is only one unique way r^k can be expressed as linear combinations of a^i's. If $m < n$, as assumed in this paper, there are more than one set of good productions ($B^k t^k = q^k$) which can exhaust the given factor supplies ($B^k r^k = f^k$) of the country at the given (p, w) and at the given coefficients of production. Therefore, we can imagine many K_q's,

[7]Here p is an n-dimensional column vector and w is an m-dimensional column vector representing the prices of goods and factors respectively. Competitive equilibrium is characterized by the profit condition. Profits are defined for the ith activity as $p_i q_i - \sum_j w_j f_j^i$ where $f^i = q_i a^i$. It is assumed that anyone who governs an activity tries to maximize profits, but since the activities may be operated at any scale, if an activity is used, it must realize zero profits. Otherwise there would be no limit to its size.

[8]A country is said to be "specialized" if it must use fewer independent activities than it has ultimate factors, i.e., if r^k lies on the boundary of K_{pw}. And of course the rank of A is assumed to be not less than m in this paper.

the region of K_{pw} possibly spanned by the productive activities actually used in the country.

A given level of factor endowments f^k of a country implies a given level of per capita income for the country. Suppose there is a given n-dimensional

$$\sum_{i=1}^{n} c_i = 1$$

row vector of final demand for each good, $x = (x_1, ..., x_n)$, for a given level of

$$\left(\text{for } 0 \le c_i \text{ and } \sum_{i=1}^{n} c_i = 1\right). \quad \text{And } r^0 = \sum_{i=1}^{n} c_i a^i$$

per capita income of a country. We select the vector for which
to represent the unit level of demand. That is, $x = B^0 c$ where B^0 is a nonnegative constant
is an m-dimensional row vector which represents the factor supply ratios actually required to satisfy the domestic demand for the country. If there were no trade, r^0 should equal r. However, with trade, r^0 need not equal r; a country may assume any set of t_i which can exhaust r, i.e. $tA = r$, and then trade with other countries to obtain the collection of goods (x) whose factor proportions requirement can be expressed as r^0, subject to the fixed maximum amount of total income ($Y = p \cdot q = p \cdot x$) attainable in equilibrium with given factor endowment f. The competitive equilibrium with equalized factor prices implies that r^0 of each country is also in K_{pw}.

3. SPECIFICATION OF THE EQUILIBRIUM PATTERN OF PRODUCTIN AND TRADE

Trade pattern of a country is determined by the difference between its internal demand pattern and production pattern. Since there are more than one q (= Bt) vectors which can exhaust the given factor endowments of a country if $m < n$, even if we were given a definite demand pattern (x) corresponding to the specific factor endowments of a country, we cannot have a definite idea about what production and trade pattern a country would assume.

In order to determine the exact q of a country and at the same time to obtain a model which can specify the pattern of production and trade on the

basis of internal demand pattern, we will introduce the following assumption: there is in each country the tendency to eliminate from trade those goods which are available through domestic production, domestic producers satisfying domestic consumption as much as possible with available productive resources. That is, we are assuming that each country tends to minimize its international transaction activities. With this assumption, as will be shown, we can obtain a unique global solution of the model with respect to the pattern of production and trade by specifying the internal demand patterns of the countries. Of course, this is still far from the world visualized by Linder. But with a further modification in the next section, the world approximated by our model comes fairly closely to the Linder's.

Before proceeding, we need a further discussion on our minimum-value-of-trade assumption. This assumption is in a sense equivalent to assuming that it is the producers who adjust in such a way that the value of foreign trade is minimized, while the consumption demand is unchanged. This would mean that the consumers pay the same price for a given good whether it is imported or domestically produced. (Uncertainty in connection with foreign trade thus would concern the producers only.) Suppose a world of two countries with identical factor endowments (and hence identical per capita income and demand pattern). With the minimum-value-of-trade assumption, there would be no trade; we have implicitly introduced the trade barrier in the form of producer's trade-minimizing activities. Linder, however, slips in an entirely opposite proposition, i.e., the amount of trade should be maximum because "there is no difference between trade among countries with the same per capita income and trade within a country."[9] Ironically, his proposition is in a sense a too literal acceptance of the traditional assumption of no-trade-barrier-whatsoever in the pure theory of international trade. If this is really the case, it is a one-country world, not two, in any economically meaningful sense. Furthermore, Linder's proposition is self-contradictory to his initial emphasis of "the world of imperfect knowledge," "unfamiliarity with foreign markets" and "the significance of internal demand structure."[10] With his proposition, the determination of the pattern of production and trade becomes a purely random process, and hence, Bhagwati comments, Linder "begins by

[9]Linder, *op. cit.*, p. 102.

[10]*Cf.* "Once we push the assumption of zero transport costs to its ultimate limit, there is absolutely no localization of demand, and there can be no a priori correlation between regional patterns of production and tastes." Quote from P. A. Samuelson, "The Transfer Problem and Transport Costs," in *Readings in International Economics*, ed. by R. E. Caves and H. G. Johnson (Homewood: Irwin, 1968), p. 125.

[11]Bhagwati, *op. cit.*, pp. 182-184.

attempting to explain the pattern of trade, but ends up by doing something quite different."[11] Linder himself does not quite see why his proposition is self-contradictory to the world visualized by himself. But it seems clear that the implicit recognition, rather than the literal ignorance, of trade barriers among countries is logically consistent with his initial emphasis of the world of imperfect knowledge, and at the same time can save his ingenious attempt to explain patterns of production and trade from ending up with a mere hypothesis concerning the volume of trade.

Now we return to the problem. In the factor price equalization theorem, regardless of the indeterminacy of international specialization, a set of imputed world good and factor prices emerges for the given production technology, factor endowments and demand structures of each country. We will have a set of (p, w) which will make a competitive equilibrium in the world possible. Therefore, the problem presented here is easily transformed to one which minimizes a linear form of nonnegative variables subject to a

$$V_k = \sum_{i=1}^{n} \left| p_i x_i - p_i q_i \right| = \sum_{i=1}^{n} \left| p_i B^0 c_i - p_i B^k t_i \right|$$

system of linear equations for the given set of (p, w). That is, we can consider the problem in the following form: find the values of $t_1, \ldots, t_n$ for kth country which minimizes the linear form
subject to the condition that $0 \leq t_i$ and $r = tA$, where p_i, A, r, x_i, c_i, B^k and B^0 are given constants to the kth country ($j = 1, \ldots, m$; $k = 1, \ldots, N$).[12]

4. MAGNITUDE OF INTERNAL DEMAND

Linder's first proposition is that the precondition for a good to be produced domestically is the presence of "home demand." He reasons that if for some reason a producer decided to cater for a demand which did not exist at home, he would probably be unsuccessful as he could not have easy access to crucial information which must be funnelled back and forth between producers and consumers, and as the producer would be unfamiliar with the conditions under which his product would have to be used. In a technical

[12]For further discussion on the formulation of the "linear form" which is to be minimized here, refer to Wontack Hong, "The Heckscher-Ohlin Theory of Factor Price Equalization and the Indeterminacy in International Specialization," *International Economic Review* (forthcoming).

sense, it may be possible to solve all problems without such close contact. But, although a producer lacking testing ground at home may be able to secure the necessary information, he would incur additional costs which might be destructive to his efforts to use the most efficient production function available in the world. Furthermore, the ability to perceive the problem of producing any particular good is likely to be generated by clearly discernible economic needs of which the producers are aware, and these would tend to arise from domestic needs.[13]

Now the factor price equalization theorem assumes the same production functions in every country, i.e., every country is assumed to use the most efficient production technology available to produce each good. We can see the problem in another way. If a country does not possess "the ability to use the same international production function for a certain good," that good will not be produced in the country. The "ability . . ." itself might be regarded as a factor of production; the production activities being limited by the supply of this factor along with other factors. But here we first assume simply that a country can have "the ability . . ." if there is domestic demand for the good. It implies that a particular good will not be produced domestically at the same efficiency (i.e., the same international production function will not be used) unless there is an internal demand for the good. Therefore, a sector will be empty if there is no internal demand, i.e., $a_{ij}^k = 0$ and $b_i^k = 0$ for some i ($i = 1, \ldots, n$). We are assuming that a producer in a country will be able to use the most efficient production function available if there is a domestic demand for the good, and once he succeeds in using the same international production function, he will be able to survive the competition in both his domestic and foreign markets. The process might be as described by Linder:

> As a successful firm grows, the local market becomes insufficient for further expansion. The trade horizon of the firm is gradually lifted. But, only after what has probably been a considerable period of producing for the domestic market will the entrepreneur become aware of the profit opportunities offered by producing for foreign countries. The export market will not be entered until then.[14]

Now we come to Linder's second proposition: a particular good will not be produced at a comparative advantage unless the internal demand for the good is representative (or large enough). So far we have assumed that a country will be able to use the most efficient production function available if there is a

[13]Linder, *op. cit.*, pp. 87-89.
[14]*Ibid.*, p. 88.

domestic demand for the good. And we have also assumed that a production technology is characterized by a linear and homogeneous production function regardless of the scale of production. But there is no question that the magnitude of domestic demand itself is very important for the start of domestic production. It is well known that indivisibilities play a role in the process of economic growth and it is a fact that some investment projects make sense only when they have a certain minimum size. The model presented so far does not show this feature, since it would introduce a non-linear relationship which would be more difficult to handle mathematically. However, we can approach the problem with a set of piecewise linear production functions representing increasing marginal returns, but still maintaining the linear homogeneity property. Suppose that for every i and every j ($i = 1, \ldots, n$; $j = 1, \ldots, m$) where is the quantity of jth factor used to produce one unit of ith good up to a certain scale of production of ith good, say min (x_i), and f_{ij} is that of ith good beyond that minimum scale of operation. If the domestic demand for certain good is less than the specified minimum amount, min (x_i), which may be different for each good, then even though the "possibility" of its domestic production might be "perceived," its domestic production would not be started because it would not be able to survive in the competition with foreign producers in the beginning period when the scope of domestic producers is limited to the local market. Now so long as we assume that there exists some other country (or countries) in which domestic demand for the ith good is greater than min (x_i), no production activity for the ith good will be carried in the country where the internal demand for ith good is less than min (x_i) because it simply violates the efficiency condition, and the production activity for the ith good will be carried out in those other countries. Hence we can expect that minimum efficient scale argument would not change the basic solutions of the factor price equalization theorem in any way, but would contribute to more realistically narrow down the range of possible productive activities of each country. It also conforms to Linder's intention to narrow down the range of potential exports by introducing the concept of "representative demand." Basically, Linder's world is regarded here as a subset of the worlds which could be generated by the Heckscher-Ohlin theory of factor price equalization.

If the magnitude of domestic demand for some goods are less than the amount required for minimum efficient scale of production, then the range of q^k would be limited by this absence of adequate amount of domestic demand, i.e., we assume that the (the region of S spanned by the activities actually

used in the country when the world output is q^w—the superscript w representing world) cannot include those processes whose domestic demands are less than certain minimum amounts. If there is systematic relationship between the demand pattern and factor supply ratios (and hence the per capita income level), we would be able to forecast the possible range of q according to the level of economic growth of a country. Linder has intended to predict the pattern of production and trade using the concept of representative demand, and he thinks that it would be easy to determine which goods are representative of the demand structure at a given per capita income. We have used the concept of minimum efficient scale in this section. Although it cannot serve as a perfect substitute for the concept of representative demand, if we apply a broader interpretation to the minimum scale concept, the result would approximate closely the world visualized by Linder.

We can now summarize the arguments: for a given set of production technology and prices (p, w), the pattern of production and trade of each country will be determined in such a fashion as to select a t vector which minimizes V subject to the condition that $r = tA$ within the range set by the magnitude of domestic demand for each good (i.e., $t_i = 0$ if $x_i < \min(x_i)$ for some i where $i = 1, \ldots, n$).

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